Assignment 6

Exponents and Radicals; Logarithms

Textbook Assignment: Chapters 7 (77-79), 8 (80-86)

6-1. The square root of 7 is an example of an irrational number.

6-2. Rationalizing the denominator is the process whereby an irrational number in the denominator of a fraction is changed to a rational number.

6-3. To rationalize the denomiantor of $\frac{4}{\sqrt{3}}$, multiply both numerator and denominator by

1. 1
2. 4
3. $\sqrt{3}$
4. $\frac{4}{\sqrt{3}}$

6-4. To rationalize the denominator of $\frac{5\sqrt{2}}{3\sqrt{3}}$, multiply both numerator and denominator by

1. $\frac{\sqrt{3}}{\sqrt{2}}$
2. $3\sqrt{3}$
3. $\sqrt{3}$
4. 1

6-5. What is the form of the fraction $\frac{3}{2\sqrt{5}}$ after the denominator has been rationalized?

1. $\frac{3\sqrt{5}}{10}$
2. $\frac{3\sqrt{5}}{2}$
3. $\frac{15}{2}$
4. $\frac{3}{10}$

6-6. What is the proper way to group the digits of the number 418.796 when preparing to calculate its square root?

1. 418. 796
2. 4' 18.7' 96
3. 41' 8.7' 96
4. 04' 18. 79' 60

6-7. In calculating the square root of 4,096, the first digit in the answer is the greatest number whose square is contained in 40, that is, the square is either equal to 40 or is less than 40.

● Note that in the square root process each trial division is obtained by multiplying the quotient by 20. For example,

```
                2   7.  1  6
               √7' 38. 00 00
                4
   20.2 = 40 | 3  38
          47 | 3  29
   20.27 = 540   |  9  00
           541   |  5  41
   20.271=5420   |  3  59  00
          5426   |  3  25  56
                       33  44
```

Therefore, $\sqrt{738} = 27.2$ (rounded to tenths)

6-8. What is the square root of 324?

1. 17.62
2. 17.94
3. 18.00
4. 22.00

6-9. In the following problem the process of taking the square root is correct to the point to which it has been carried.

```
          7 1 .
         √63'83.09
   140    49
   141   148
         141
           7
```

6-10. What is the error in the following square root calculation?

```
          9  4.0
         √88'20.00
          81
   180    720
          720
```

1. The trial divisor was not adjusted to form a true divisor.
2. The digits were not properly grouped.
3. There is an error in multiplication.
4. The decimal point is not properly alined.

6-11. The decimal point in a square root calculation is kept alined as in long division with the exception that alinement is accomplished with pairs of digits rather than with single digits.

6-12. What is the square root of 15,129?
1. 102.3
2. 123
3. 390.1
4. 393

6-13. What is the square root of 816.7 correct to the nearest tenth?
1. 9.0
2. 28.5
3. 28.6
4. 29.9

6-14. If the square root of 54 is 7.35, the square root of 5,400 is 73.5.

6-15. If the square root of 3,812 is 61.741, the square root of 38,120 is 617.41.

6-16. If the cube root of 89 is 4.46, the cube root of 0.089 is
1. 0.00446
2. 0.0446
3. 0.446
4. 44.6

6-17. In the expression $3^4 = 81$, which number may be interpreted as a logarithm?
1. 3
2. 4
3. 64
4. 81

6-18. What is the logarithmic form of the expression, $2^5 = 32$?
1. $\log_2 5 = 32$
2. $\log_2 32 = 5$
3. $\log_5 32 = 2$
4. $\log_{32} 5 = 2$

● Refer to table 8-1 in your textbook in answering items 6-19 and 6-20.

6-19. What base is used in the system of logarithms in which the logarithm of the number 16 is 1?
1. 1
2. 2
3. 4
4. 16

6-20. What is the value of x if $\log_3 9 = x$?
1. 2
2. 3
3. 9
4. 27

6-21. Since a logarithm is an exponent, multiplication using logarithms is reduced to a problem of addition of logarithms.

6-22. Refer to table 8-2 in your textbook. Which of the following is correct in the multiplication of 16 x 128?
1. $\log_4 16 = 2$
   $\log_2 128 = 7$
   $\log_2$ of the product = 9
2. $\log_2 16 = 4$
   $\log_2 128 = 7$
   $\log_2$ of the product = 11
3. $\log_2 16 = 4$
   $\log_2 128 = 7$
   $\log_2$ of the product = 28
4. There is not enough information given in the table to work this problem.

6-23. What number is used as the base of the system of logarithms for most ordinary computations?
1. 2
2. 2.3026
3. 2.71828
4. 10

6-24. When the word log is used without a subscript, it is understood that the base 10 is to be used.

6-25. Assume that you have used a formula involving natural logarithms and the answer you have found is $\ln x = 0.29366$. You can find the value of x by first applying the correct conversion factor to obtain
1. $\log x = 0.123582$
2. $\log x = 0.127537$
3. $\log x = 0.158243$
4. $\log x = 0.675416$

● The characteristic of a number may be determined by writing the number in scientific notation. The resulting exponent is the characteristic. For example, in log .0078, write .0078 as 7.8×10^{-3}. The characteristic is then -3. For log 256, write 256 as 2.56×10^2. The characteristic is then 2.

● Items 6-26 through 6-60 refer to common logarithms unless otherwise indicated.

6-26. What is the log of 0.00001?
1. -5
2. -4
3. -3
4. $\frac{1}{-5}$

6-27. What is the common logarithm of 100,000?
1. 3
2. 5
3. 7
4. 10

6-28. The log of a number between 100 and 1,000 is between
1. 0 and 1
2. 1 and 2
3. 2 and 3
4. 3 and 4

6-29. Refer to table 8-3 in your textbook. Between what logarithms may the logarithm of 0.0004 be located?
1. -2 and -3
2. -3 and -4
3. -4 and -5
4. 3 and 4

6-30. If $2 = 10^{0.30103}$, and $2 \times 5 = 10^{1}$, to what power must 10 be raised to equal 5?
1. 0.47712
2. 0.60206
3. 0.69897
4. 0.90309

6-31. If the characteristic of a logarithm is 1, the associated number must be between
1. 0 and 1
2. 1 and 10
3. 10 and 100
4. 100 and 1,000

6-32. For any number greater than 1, the characteristic is one less than the number of digits in the whole number portion of the number.

6-33. What is the characteristic of 72,319?
1. 3
2. 4
3. 5
4. 6

6-34. Which of the following numbers has a characteristic of -4?
1. 0.00001
2. 0.00095
3. 0.10005
4. 0.40008

6-35. What is the value of the mantissa in the expression log 0.0054 = 7.73239 - 10?
1. -3.73239
2. 0.0054
3. 0.73239
4. 7.73239

6-36. The mantissa for the numerical sequence 165 is 0.21748. Which of the following logarithms can be used to express the decimal fraction 0.000165?
1. $\bar{4}$.21748
2. 0.21748 - 4
3. 6.21748 - 10
4. All of the above

6-37. If the mantissa for the number sequence 17,900 is 0.25285, what is the log of 179?
1. 1.25285
2. 2.25285
3. 4.25285
4. 8.25285 - 10

● In answering items **6-38** through **6-41**, refer to the table of logarithms in Appendix I.

6-38. What is the log of 70?
1. 0.1213
2. 1.1213
3. 1.84510
4. 2.84510

6-39. What is the log of 2,700?
1. 0.43136
2. 1.43136
3. 2.43136
4. 3.43136

6-40. What is the log of 0.0024?
1. 0.38021
2. 7.38021 - 10
3. 8.38021 - 10
4. 9.38021 - 10

6-41. What is the log of 1?
1. 0.00000
2. 0.10000
3. 0.10000 - 10
4. 9.00000 - 10

● An antilogarithm is a number which corresponds to a logarithm; for example, in log 5.2 = .716, 5.2 is said to be the antilogarithm of .716. Mathematically, antilog .716 = 5.2. Generalizing, for log N = L; N is the antilogarithm and L is the logarithm. Finding the antilogarithm is the reverse process of finding the logarithm, that is, rather than determining the charateristic and mantissa of a number, the number must be determined given the characteristic and mantissa.

EXAMPLE: Find the antilogarithm of 2.9345

SOLUTION:

1. Find the mantissa .9345 in column six of Appendix I. This mantissa corresponds to the digit sequence 86.

2. Since the characteristic of the original logarithm is 2 then the antilogarithm written in scientific notation is 8.6×10^2 or antilog 2.9345 = 860

6-42. If log 12 = 1.07918 then the antilog equals
1. .3333
2. 1.07918
3. 10
4. 12

● Refer to Appendix I in answering items 6-43 and 6-44.

6-43. If log A = 1.83251 then A equals
1. .3010
2. 6.8
3. 30.10
4. 68

6-44. If antilog 3.62325 = B then B equals
1. .5563
2. 420
3. 4200
4. 5563

● The logarithm of a product is equal to the sum of the logarithms of the factors, that is

$\log (a \cdot b \cdot c \cdot d) = \log a + \log b + \log c + \log d$

EXAMPLE: Find the product of $3 \cdot 4$ using logarithms.

SOLUTION:

1. $\text{Log } (3 \cdot 4) = \log 3 + \log 4 = .47712 + .60206 = 1.07918$
2. Antilog 1.07918 = product, or antilog 1.07918 = $1.2 \times 10^1 = 12$
Therefore $3 \times 4 = 12$.

6-45. Since log 40 = 1.60206 and log 5 = .69897 , the log of the product of 40 x 5 is equivalent to
1. log 40 x log 5
2. 1.60206 x .69897
3. 1.60206 + .69897
4. log 1.60206 + log .69897

● Refer to Appendix I in answering items 6-46 through 6-48

6-46. Use logarithms to find the product of 28 x 20. What is the mantissa which must be used to find the digit sequence for the product?
1. .27481
2. .30103
3. .74819
4. 1.30103

6-47. The antilog of what value must be used to find the product of 59 x 38 ?
1. 3.35063
2. 3.77085
3. 4.35063
4. 4.77085

6-48. Use logarithms to find the product of 2900 and 3000. The product equals
1. 7.7×10^6
2. 8.7×10^6
3. 7.7×10^7
4. 8.7×10^7

● The logarithm of a power of a number is equal to the product of the power and the logarithm of the number. That is, $\log A^n = n \log A$.

Note that $A^4 = A \times A \times A \times A$

and $\log A^4 = \log A + \log A + \log A + \log A$

[taking logarithms of both sides of the equations] or $\log A^4 = 4 \log A$

EXAMPLE: Find the value of 3^4, using logarithms.

SOLUTION:

1. $\text{Log } 3^4 = 4 \log 3 = 4(.47712) = 1.90848$

2. Antilog 1.90848 = answer, or antilog 1.90848 = $8.1 \times 10^1 = 81$

6-49. $\text{Log } 16^{32}$ is equal to which of the following?
1. log 16 + log 32
2. log 16 x log 32
3. 16 log 32
4. 32 log 16

6-50. Using logarithms, find the approximate value of 5^8 .
1. 3.9×10^5
2. 3.9×10^6
3. 4.0×10^5
4. 4.0×10^6

6-51. The $\log 26 \cdot 38^4$ is equivalent to
1. 104 log 38
2. log 26 x 4 log 38
3. log 26 + log 4 + log 38
4. log 26 + 4 log 38

6-52. The antilog of what value is used to find the approximate product of $22 \cdot 43^6$? (Refer to Appendix 1)

1. 10.14324
2. 11.14324
3. 12.15683
4. 13.15683

● Note that
$$\log \frac{a}{b} = \log a \cdot b^{-1}$$
$$= \log a + \log b^{-1}$$
$$= \log a - 1 \log b$$
$$= \log a - \log b$$

Therefore, the logarithm of the quotient of two numbers equals the logarithm of the dividend minus the logarithm of the divisor.

EXAMPLE: Find the value of $\frac{24}{8}$ using logarithms.

SOLUTION:

1. $\text{Log } \frac{24}{8} = \log 24 - \log 8$
$= 1.38021 - .90308$
$= .47713$

2. Antilog .47713 = quotient, or antilog .47713 = $3 \text{ X } 10^0$
$= 3 \text{ X } 1 = 3$
Therefore $\frac{24}{8} = 3$

6-53. Since log 12 = 1.07918 and log 2 = .30103, the log of the quotient of $\frac{12}{2}$ is equivalent to

1. .30103 - 1.07918
2. 1.07918 - .30103
3. log 1.07918 ÷ log .30103
4. log 12 ÷ log 2

● Refer to Appendix I in answering items 6-54 through 6-56.

6-54. Use logarithms to find the quotient of $\frac{81}{45}$. What is the mantissa used to find the digit sequence of the quotient ?

1. .25528
2. .30103
3. .36000
4. .56170

6-55. The antilog of what value must be used to find the quotient of $\frac{540}{36}$?

1. 1.28869
2. 1.17609
3. 3.17609
4. 3.28869

6-56. Use logarithms to find the quotient of $\frac{780000}{30}$. The quotient equals

1. $2.3 \text{ X } 10^3$
2. $2.6 \text{ X } 10^3$
3. $2.3 \text{ X } 10^4$
4. $2.6 \text{ X } 10^4$

6-57. $\text{Log } \frac{22^3}{6}$ is equivalent to

1. 3 log 22 - 6
2. 3 log 22 - log 6
3. log 3 X log 22 - log 6
4. log 22 + log 3 - log 6

6-58. Which expression below is equivalent to $\log \frac{.5(14^5)}{(6)(13^2)}$?

1. 5 log .5 + log 14 - log 6 X 2 log 13
2. log .5 +5 log 14 - log 6 X 2 log 13
3. log .5 x5 log 14 - (log 6 + 2 log 13)
4. log .5 +5 log 14 - (log 6 + 2 log 13)

6-59. The expression $\log \frac{4}{7^{-3}}$ is equivalent to

1. $4 \log 7^{-3}$
2. log 4 - 3 log 7
3. log 4 + 3 log 7
4. 3 log 7 - 4

6-60. The antilog of what value is used to find the approximate result of $\frac{13 \cdot 4^2}{7}$? (Refer to Appendix I)

1. .63132
2. 1.47296
3. 1.63132
4. 2.47296